Complexity in strongly correlated systems

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(Do not try this at home)

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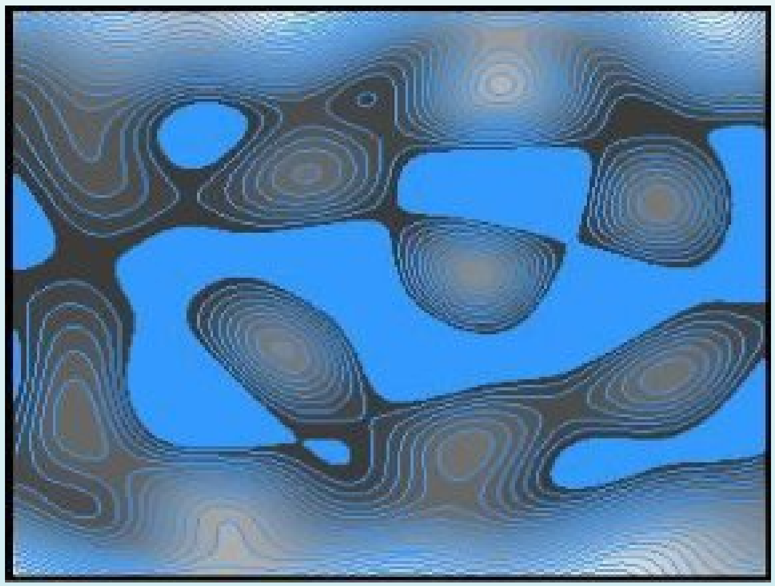
System	Name	Use
2D electrons in magnetic field	Quantum Hall droplets	quantum information
2D frustated spin systems	glassy states	information theory
1D interacting electrons	nanostructures	nanotechnology
1D quantum magnets	spin chains	entanglement

Quantum Hall droplets



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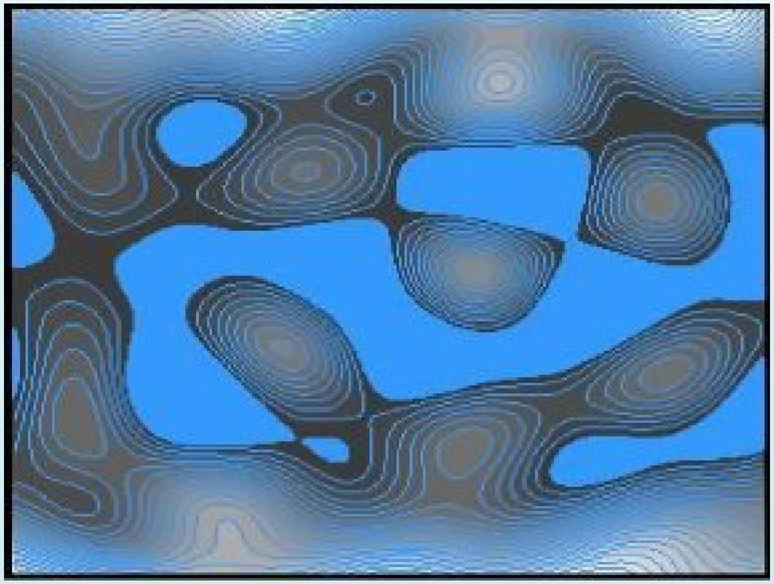
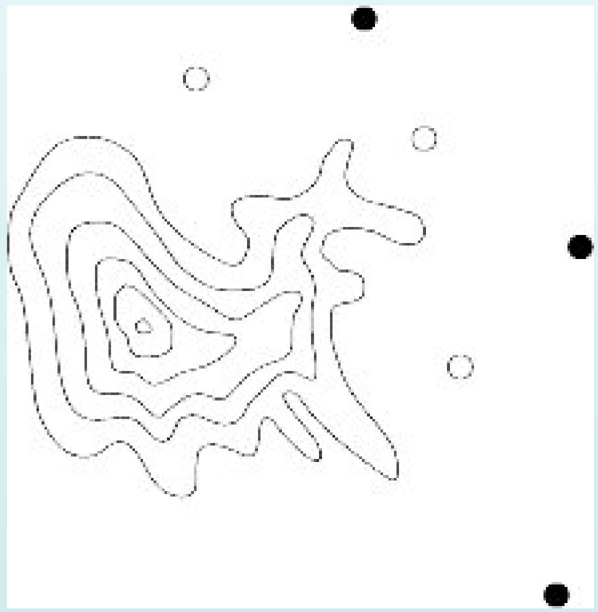


2D electrons in transversal magnetic field, and confining electrostatic potential (1985 and 1998 Nobel prizes)

Incompressible fluids ... with a twist



Elementary magnetic fluxes (tubes) infinitesimally deform the droplet.

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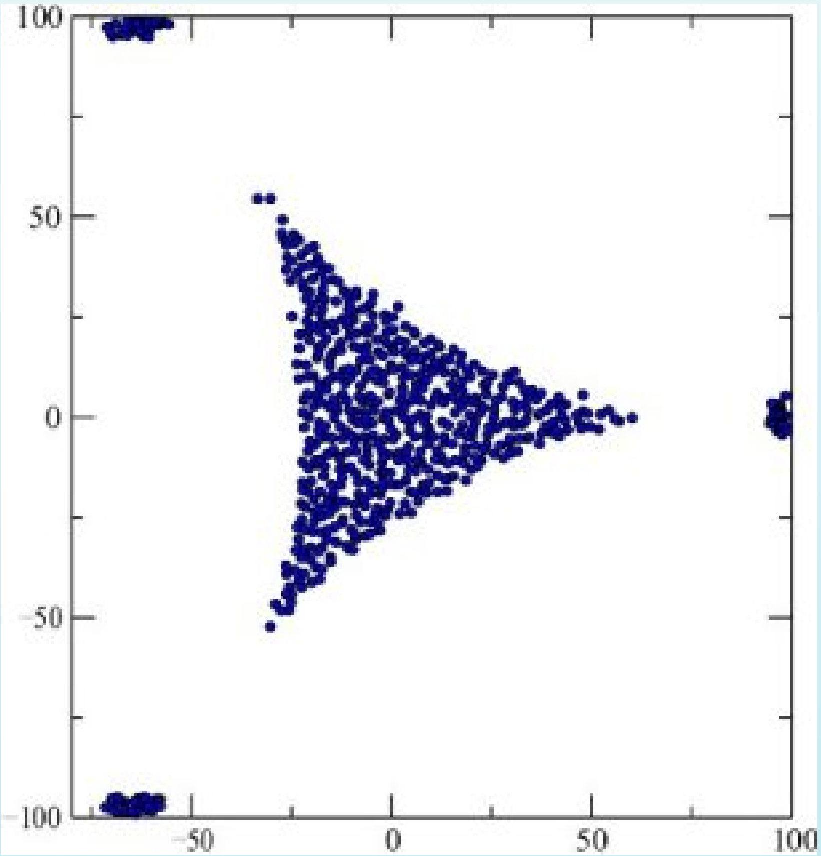
- Problem becomes "stochastic electrostatics"

Quantum droplets as Coulomb charges



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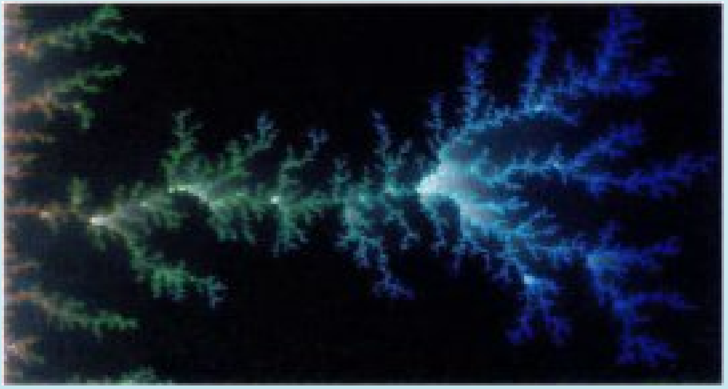
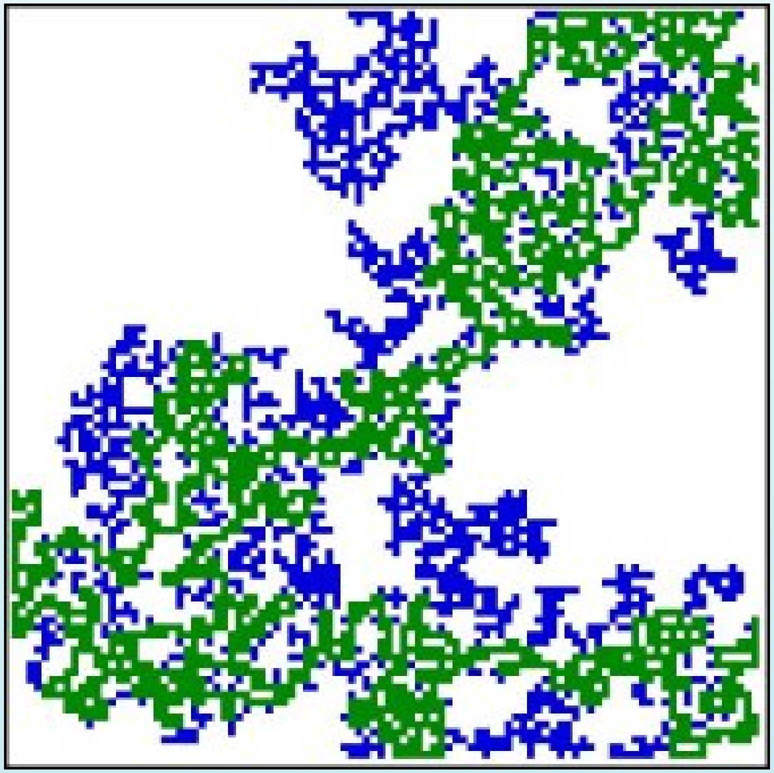


2D spin systems



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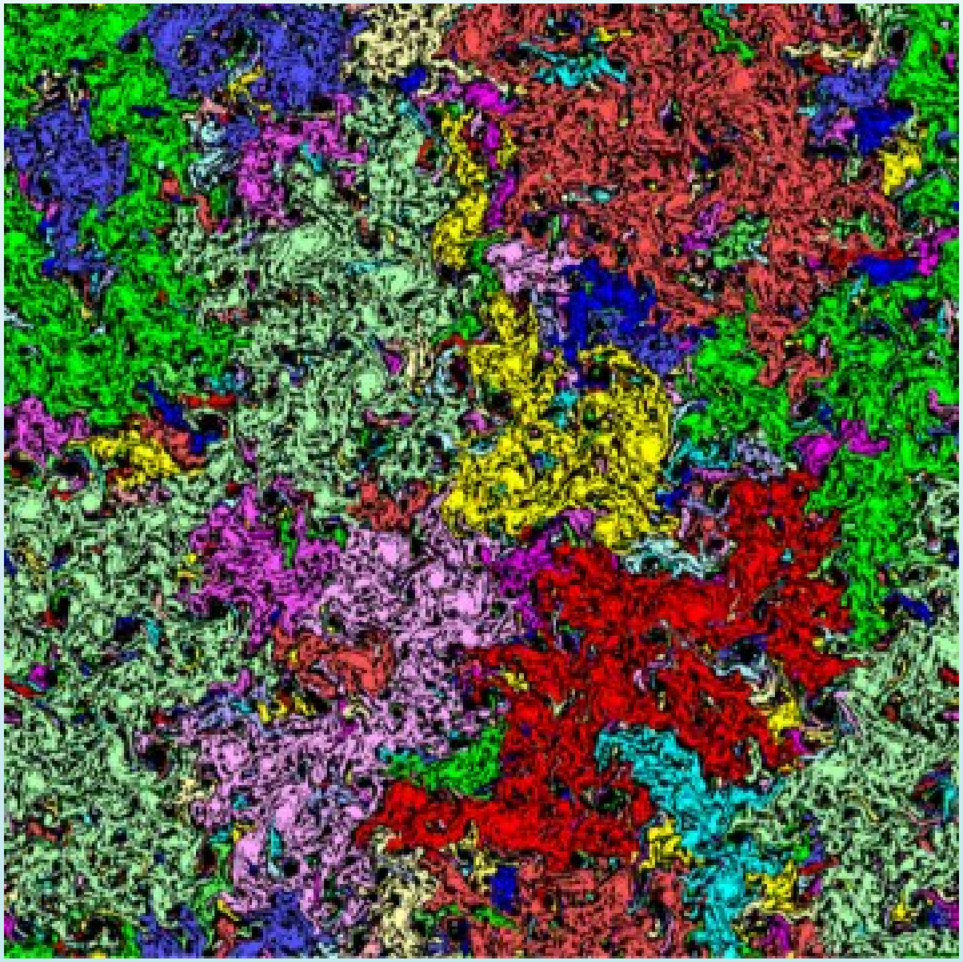
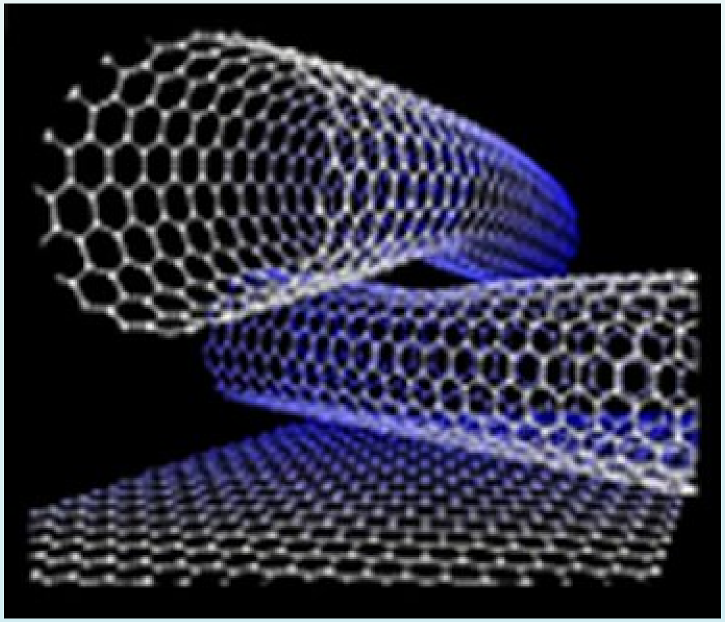
Spin systems and hydrodynamics



Nanostructures: 1D quantum hydrodynamics



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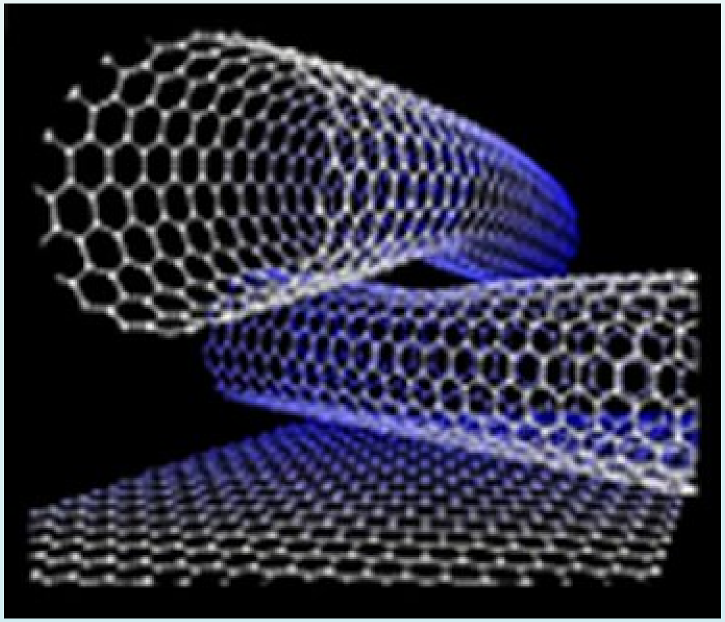

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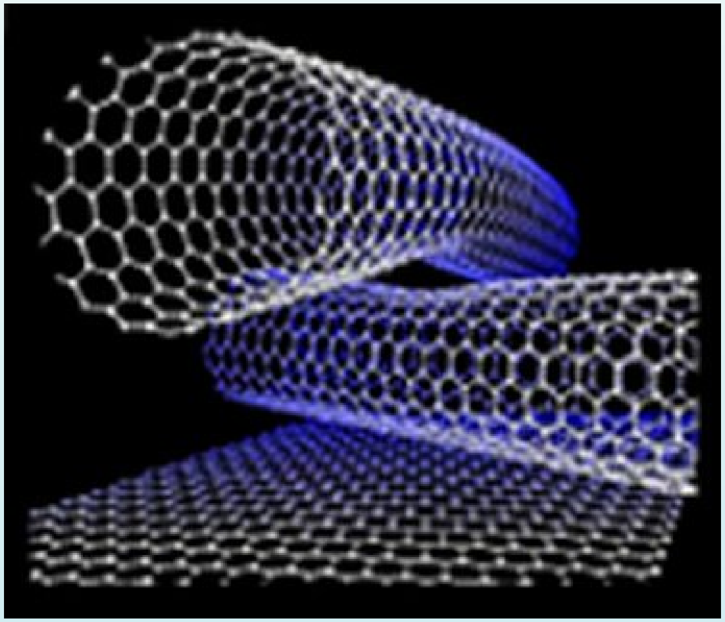

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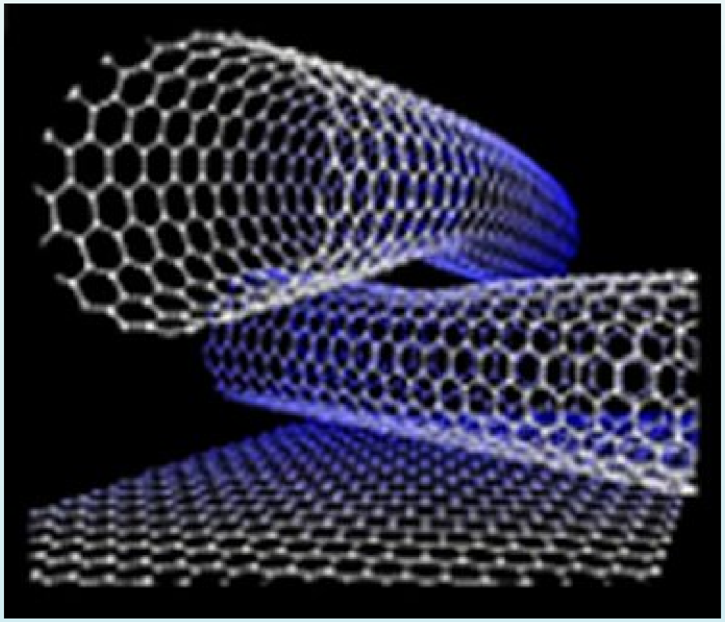

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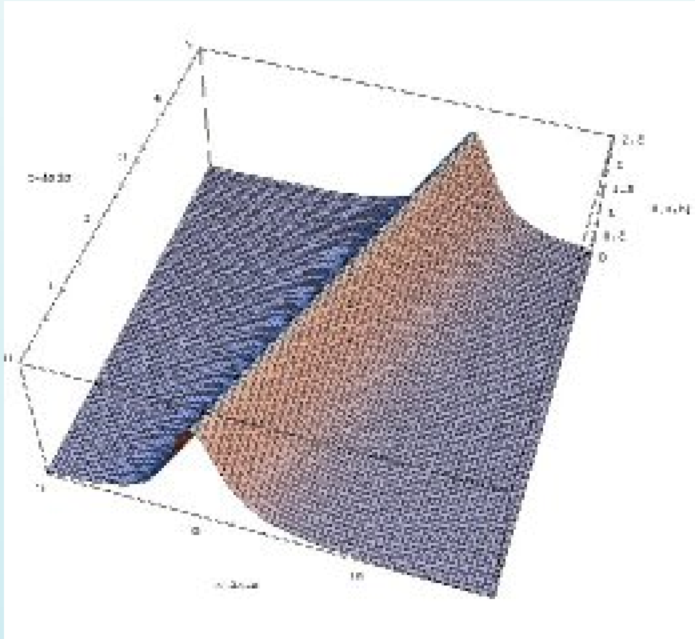

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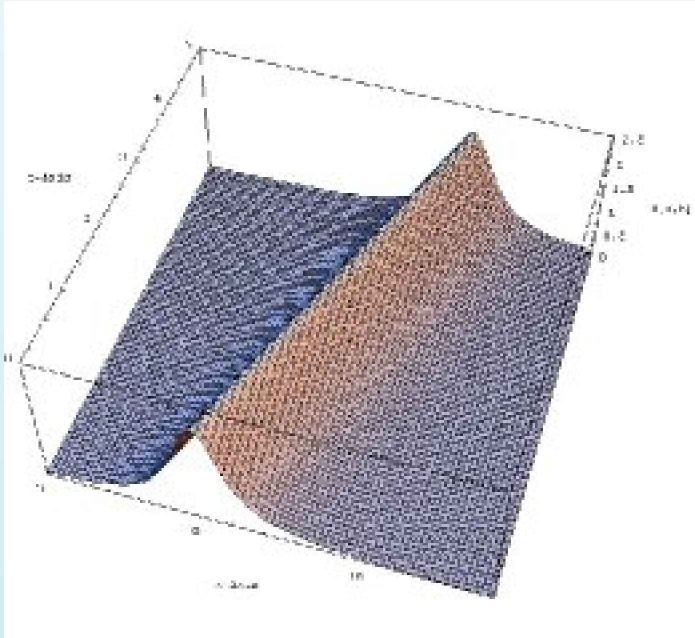

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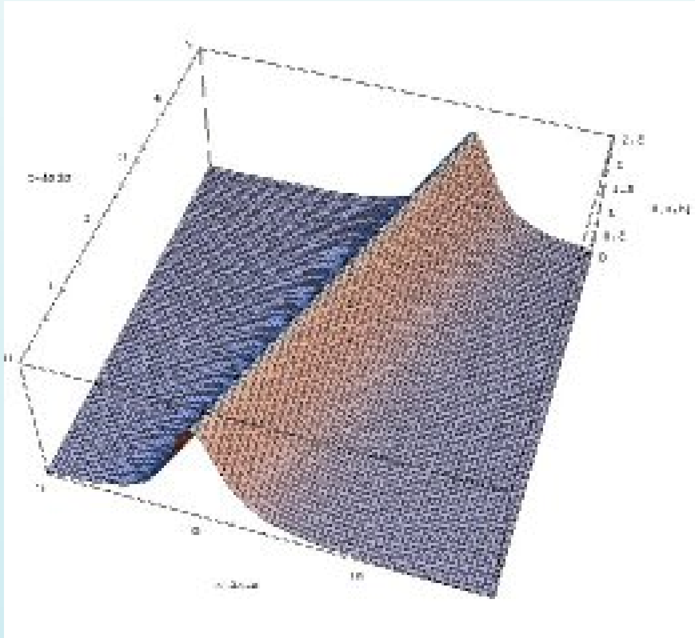

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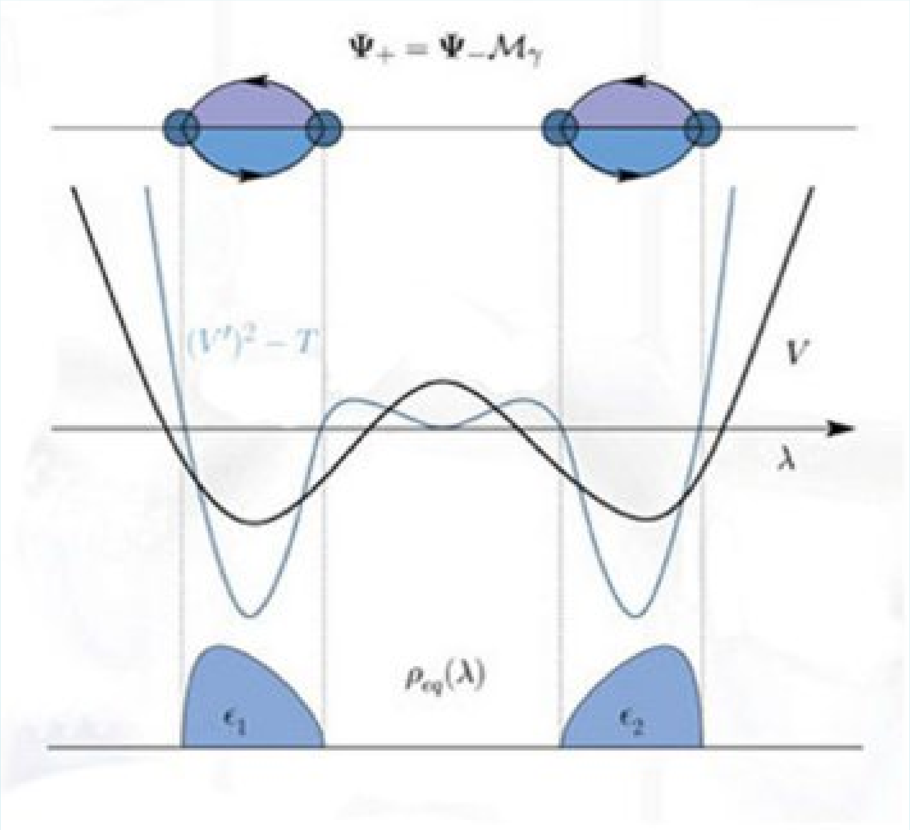
Entanglement in 1D – free boundary again !



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